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General Certificate of Education

Mathematics 6360

MPC3 Pure Core 3

Mark Scheme

2010 examination - January series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Set and published by the Assessment and Qualifications Alliance.

Key to mark scheme and abbreviations used in marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√ or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
$-x$ EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

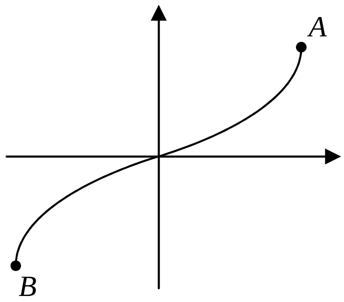
Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

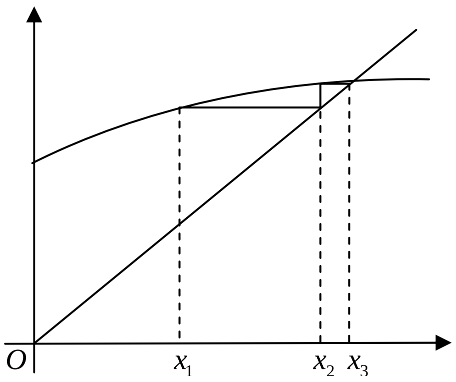
MPC3

Q	Solution	Marks	Total	Comments
1(a)	$y' = e^{-4x}(2x+2) \quad -4e^{-4x}(x^2+2x-2)$	M1		$y' = Ae^{-4x}(ax+b) \pm Be^{-4x}(x^2+2x-2)$ where A and B are non-zero constants
		A1		All correct
	$= e^{-4x}(2x+2-4x^2-8x+8)$			**or** $-4x^2e^{-4x} - 6xe^{-4x} + 10e^{-4x}$
	$= 2e^{-4x}(5-3x-2x^2)$	A1	3	AG; all correct with no errors, 2nd line (OE) must be seen Condone incorrect order on final line
	or $y = x^2 e^{-4x} + 2xe^{-4x} - 2e^{-4x}$			
	$y' = -4x^2e^{-4x} + 2xe^{-4x} + 2x. - 4e^{-4x}$ $+ 2e^{-4x} + 8e^{-4x}$	(M1) (A1)		$Ax^2e^{-4x} + Bxe^{-4x} + Cxe^{-4x} + De^{-4x} + Ee^{-4x}$ All correct
	$= -4x^2e^{-4x} - 6xe^{-4x} + 10e^{-4x}$ $= 2e^{-4x}(5-3x-2x^2)$	(A1)		AG; all correct with no errors, 3rd line (OE) must be seen
(b)	$-(2x+5)(x-1)(=0)$	M1		OE Attempt at factorisation $(\pm 2x \pm 5)(\pm x \pm 1)$ or formula with at most one error
	$x = \frac{-5}{2}, 1$	A1		Both correct and no errors SC $x = 1$ only scores M1A0
	$x = 1,\ y = e^{-4}$	m1		For $y = ae^b$ attempted
		A1F		Either correct, follow through only from incorrect sign for x
	$x = -\frac{5}{2},\ y = e^{10}\left(-\frac{3}{4}\right)$	A1	5	CSO 2 solutions only Note: withhold final mark for extra solutions Note: approximate values only for y can score m1 only
	Total		**8**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
2(a)(i)		B1		correct shape passing through origin and stopping at *A* and *B*
	$A\left(1, \frac{\pi}{2}\right)$	B1		
	$B\left(-1, -\frac{\pi}{2}\right)$	B1	3	SC $A(1, 90)$ and $B(-1, -90)$ scores B1
(ii)	line intersecting their curve (positive gradient, positive *y* intercept)	M1		
	Correct statement	A1	2	one solution only, stated or indicated on sketch - must be in the first quadrant (ie curve intersects line once) Must have scored B1 for graph in (a)(i)
(b)	$\left.\begin{array}{ll}\text{LHS }(0.5) = 0.5 & \text{RHS}(0.5) = 1.1 \\ \text{LHS}(1) = 1.6 & \text{RHS}(1) = 1.3\end{array}\right\}$	M1		
	At 0.5 LHS < RHS , At 1 LHS > RHS $\therefore\ 0.5 < \alpha < 1$	A1	2	CSO
	or			
	$\text{f}(x) = \sin^{-1}(x) - \frac{1}{4}x - 1$			$\text{f}(x)$ must be defined
	$\left.\begin{array}{l}\text{f}(0.5) = -0.6 \\ \text{f}(1) = 0.3\end{array}\right\}$ AWRT	(M1)		Allow $\text{f}(0.5) < 0$ $\text{f}(1) > 0$
	Change of sign $\Rightarrow 0.5 < \alpha < 1$	(A1)		
	or			
	$\text{f}(x) = \sin\left(\frac{1}{4}x + 1\right) - x$			$\text{f}(x)$ must be defined
	$\left.\begin{array}{l}\text{f}(0.5) = 0.4 \\ \text{f}(1) = -0.1\end{array}\right\}$ Attempt	(M1)		
	Change of sign $\Rightarrow 0.5 < \alpha < 1$	(A1)		
	or			
	$\text{f}(x) = 4\sin^{-1}x - x - 4$			$\text{f}(x)$ must be defined
	$\left.\begin{array}{l}\text{f}(0.5) = -2.4 \\ \text{f}(1) = 1.3\end{array}\right\}$ attempt	(M1)		
	Change of sign $\Rightarrow 0.5 < \alpha < 1$	(A1)		

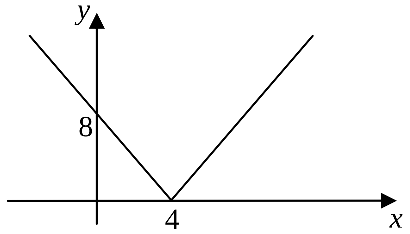
MPC3 (cont)

Q	Solution	Marks	Total	Comments
2(c)(i)	$x_2 = 0.902$	M1		Sight of AWRT 0.902 or AWRT 0.941
	$x_3 = 0.941$	A1	2	These values only
(ii)		M1		Staircase, (vertical line) from x_1 to curve, horizontal to line, vertical to curve
		A1	2	x_2 , x_3 approx correct position on x-axis
	Total		**11**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
3(a)	$\sin x = \frac{1}{3}$, or sight of ± 0.34, $\pm 0.11\pi$ or ± 19.47 (or better)	M1		
	$x = 0.34,\ 2.8(0)$ AWRT	A1	2	Penalise if incorrect answers in range; ignore answers outside range
(b)	$\text{cosec}^2 x - 1 = 11 - \text{cosec}\, x$	M1		Correct use of $\cot^2 x = \text{cosec}^2 x - 1$
	$\text{cosec}^2 x + \text{cosec}\, x - 12 (= 0)$	A1		
	$(\text{cosec}\, x + 4)(\text{cosec}\, x - 3)(= 0)$	m1		Attempt at Factors Gives cosec x or – 12 when expanded Formula one error condoned
	$\left.\begin{array}{l}\text{cosec}\, x = -4,\ 3\\ \sin x = -\frac{1}{4},\ \frac{1}{3}\end{array}\right\}$	A1		Either Line
	$\sin x = -\frac{1}{4}$			
	$\Rightarrow x = 3.39,\ 6.03$ AWRT	B1F		3 correct or their two answers from (a) and 3.39, 6.03
	$0.34,\ 2.8(0)$ AWRT	B1	6	4 correct and no extras in range ignore answers outside range SC 19.47, 160.53, 194.48, 345.52 B1
	Alternative $\frac{\cos^2 x}{\sin^2 x} = 11 - \frac{1}{\sin x}$ $\cos^2 x = 11\sin^2 x - \sin x$	(M1)		Correct use of trig ratios and multiplying by $\sin^2 x$
	$1 - \sin^2 x = 11\sin^2 x - \sin x$			
	$0 = 12\sin^2 x - \sin x - 1$	(A1)		
	$0 = (4\sin x + 1)(3\sin x - 1)$	(m1)		Attempt at factors as above
	$\sin x = -\frac{1}{4},\ \frac{1}{3}$	(A1)		
		(B1F) (B1)		As above
	Total		**8**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
4(a)		M1		Modulus graph V shape in 1st quad going into 2nd quad, touching x-axis. Must cross y-axis Condone not ruled
		A1	2	4 and 8 labelled
(b)	$x=2$	B1		One correct answer
	$x=6$	B1	2	Second correct answer and no extras Condone answers shown on the graph, if clearly indicated
(c)	$x>6$	B1		One correct answer
	$x<2$	B1	2	Second correct answer and no extras and no further incorrect statement eg $6<x<2$ or $2<x>6$ SC $x\geq 6$, $x\leq 2$ scores B1
	Total		**6**	
5(a)	$$\begin{array}{c\|c} x & y \\ \hline 1.5 & 1.98100 \\ 4.5 & 3.22883 \\ 7.5 & 4.11496 \\ 10.5 & 4.74710 \end{array}$$	B1		x values correct PI
		M1		3+ y values correct to 2sf or better or exact values
		A1		$1.981, 3.228/9, 4.114/5, 4.747$ for y (or better)
	$\int = 3\times\sum y$ $=42.2$	A1	4	(Note: 42.2 with evidence of mid-ordinate rule with four strips scores 4/4)
(b)(i)	$y=\ln\left(x^2+5\right)$ $e^y=x^2+5$			OE
	$x^2=e^y-5$	B1	1	AG Must see middle line, and no errors
(ii)	$(\pi)\int\left(e^y-5\right)(dy)$	M1		Condone omission of brackets around f (y) throughout
	$=(\pi)\left[e^y-5y\right]_{(5)}^{(10)}$	A1		
	$=(\pi)\left[\left(e^{10}-50\right)-\left(e^5-25\right)\right]$	m1		$F(10)-F(5)$
	$V=\pi\left[e^{10}-e^5-25\right]$	A1	4	CSO including correct notation – must see **d**y ISW if evaluated
(c)	$(y=)\ln\left[\left(\frac{x}{4}\right)^2+5\right]+3$	M1		$\frac{x}{4}$ seen, condone $\ln\ \frac{x^2}{4}+$ ....
		B1		… + 3
		A1	3	CSO mark final answer (no ISW)
	Total		**12**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
6(a)	$\mathrm{f}(x) > -3$	M1		'>-3', '$x>-3$' or '$\mathrm{f}(x) \geq -3$'
		A1	2	Allow $y > -3$
(b)(i)	$y = \mathrm{e}^{2x} - 3$ $y + 3 = \mathrm{e}^{2x}$			
	$\ln(y+3) = 2x$	M1		swap x and y
		M1		attempt to isolate: $\ln(y \pm A) = Bx$ or reverse
	$\left(\mathrm{f}^{-1}(x)\right) = \frac{1}{2}\ln(x+3)$	A1	3	OE with no further incorrect working Condone $y = \ldots$..
	Alternative $x \rightarrow \times 2 \rightarrow \mathrm{e} \rightarrow -3$ $\div 2 \leftarrow \ln \leftarrow +3 \leftarrow x$ (M1) (M1)			
	$y = \dfrac{\ln(x+3)}{2}$	(A1)		
(ii)	$x + 3 = 1$	M1		for putting their $\mathrm{p}(x) = 1$ from $k\ln(\mathrm{p}(x))$ in their part (b)(i)
	$x = -2$	A1	2	CSO SC: B2 x = -2 with no working, if full marks gained in part (b)(i)
(c)(i)	$(\mathrm{gf}(x) =)\ \dfrac{1}{3(\mathrm{e}^{2x}-3)+4}$ $(=)\ \dfrac{1}{3\mathrm{e}^{2x}-5}$ either OE	B1	1	substituting f into g ISW
(ii)	$\dfrac{1}{3\mathrm{e}^{2x}-5} = 1$			
	$1 = 3\mathrm{e}^{2x} - 5$ OE	M1		Correct removal of their fraction
	$\mathrm{e}^{2x} = 2$			
	$2x = \ln 2$	m1		Correct use of logs leading to $kx = \ln\dfrac{a}{b}$
	$x = \frac{1}{2}\ln 2$ OE	A1	3	CSO No ISW except for numerical evaluation
	Total		**11**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
7(a)	$\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)\frac{\cos 4x.4\cos 4x-\sin 4x.-4\sin 4x}{\cos^2 4x}$	M1		$\frac{\pm A\cos^2 4x \pm B\sin^2 4x}{\cos^2 4x}$
	$=\frac{4\cos^2 4x+4\sin^2 4x}{\cos^2 4x}$ or better	A1		Both terms correct
	$=4\left(1+\tan^2 4x\right)$ CSO	A1	3	All correct
	or			
	$\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)\frac{\cos 4x.4\cos 4x-\sin 4x.-4\sin 4x}{\cos^2 4x}$	(M1)		$\frac{\pm A\cos^2 4x \pm B\sin^2 4x}{\cos^2 4x}$
	$=\frac{4\cos 4x\cos 4x}{\cos 4x\cos 4x}+\frac{4\sin 4x\sin 4x}{\cos 4x\cos 4x}$ or better	(A1)		
	$=4\left(1+\tan^2 4x\right)$ CSO	(A1)		All correct
(b)	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=4\times 2\tan 4x\times \ldots\ldots$	M1		$A\tan 4x \times \mathrm{f}(4x)$
	$4\sec^2 4x$	m1		$\mathrm{f}(4x) = B\sec^2 4x$
	$=32\tan 4x\sec^2 4x$	A1F		ft $8\times$ their p from part (a)
	$=32\tan 4x\left(1+\tan^2 4x\right)$	m1		Previous two method marks must have been earned
	$=32y\left(1+y^2\right)$	A1	5	CSO
	Alternative Solutions			
	$y' = 4+4\tan^2 4x = 4+4\frac{\sin^2 4x}{\cos^2 4x}$			
	$y''=4\times$ $\left[\frac{\left[\cos^2 4x\,2\sin 4x 4\cos 4x+\sin^2 4x\,2\cos 4x\,4\sin 4x\right]}{\cos^4 4x}\right]$	(M1)		$\frac{A\cos^3 4x \pm B\sin^3 4x}{\cos^4 4x}$ where A and B are constants or trig functions.
		(m1)		Where A is $m\sin 4x$ and B is $n\cos 4x$
	$=\frac{4\times 8\sin 4x\cos 4x\left[\cos^2 4x+\sin^2 4x\right]}{\cos^4 4x}$	(A1F)		ft $8\times$ their p from part (a)
	$=32\tan 4x\sec^2 4x$	(m1)		$k\tan 4x\sec^2 4x$
	$=32y\left(1+y^2\right)$	(A1)		CSO
	or			
	$\frac{\mathrm{d}y}{\mathrm{d}x}=4\sec^2 4x$			
	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=4\times 2\sec 4x.4\sec 4x\tan 4x$	(M1)		$A\sec 4x \times \mathrm{f}(4x)$
		(m1)		$\mathrm{f}(4x) = B\sec 4x\tan 4x$
	$=32\sec^2 4x\tan 4x$	(A1F)		ft $8\times$ their p from part (a)
	$=32\left(1+\tan^2 4x\right)\tan 4x$	(m1)		Previous two method marks must have been earned
	$=32y\left(1+y^2\right)$	(A1)		CSO

MPC3 (cont)

Q	Solution	Marks	Total	Comments
7(b) or	$\frac{dy}{dx} = 4(1+\tan^2 4x)$			
	$u = \tan 4x \qquad \frac{dy}{dx} = 4 + 4u^2$			
	$\frac{d^2y}{dx^2} = (8)u\frac{du}{dx}$	(M1)		
	$\frac{du}{dx} = 4 + 4\tan^2 4x = 4 + 4u^2$	(m1)		
	$\frac{d^2y}{dx^2} = 8u(4+4u^2)$	(A1)		
	$= 32u(1+u^2)$	(m1)		
	$= 32y(1+y^2)$	(A1)		
	Total		**8**	
8(a)	$\int x\sin(2x-1)\,dx$			
	$u = x \quad \frac{dv}{dx} = \sin(2x-1)$	M1		$\int \sin f(x)$, $\frac{d}{dx}(x)$ attempted
	$\frac{du}{dx} = 1 \quad v = -\frac{1}{2}\cos(2x-1)$	A1		All correct – condone omission of brackets
	$(\int =) -\frac{x}{2}\cos(2x-1)$	m1		correct substitution of their terms into parts
	$-\int -\frac{1}{2}\cos(2x-1)\ (dx)$			
	$= -\frac{x}{2}\cos(2x-1) + \frac{1}{2}\int\cos(2x-1)\ (dx)$	A1		All correct – condone omission of brackets
	$= -\frac{x}{2}\cos(2x-1) + \frac{1}{4}\sin(2x-1) + c$	A1	5	CSO condone missing + c and dx Condone missing brackets around $2x - 1$ if recovered in final line ISW
(b)	$u = 2x-1$			
	$'du = 2dx'$	M1		OE
	$\int \frac{x^2}{2x-1}\,dx = \int \frac{(u+1)^2}{4u}\frac{du}{2}$	m1 A1		All in terms of u All correct PI from later working
	$= \left(\frac{1}{8}\right)\int \frac{u^2+2u+1}{u}\,du$			
	$= \left(\frac{1}{8}\right)\int u + 2 + \frac{1}{u}\,du$	A1		
	$= \left(\frac{1}{8}\right)\left[\frac{u^2}{2} + 2u + \ln u\right]$	B1		or $\left(\frac{1}{8}\right)\left[\frac{(u+2)^2}{2} + \ln u\right]$
	$= \frac{1}{8}\left[\frac{(2x-1)^2}{2} + 2(2x-1) + \ln(2x-1)\right] + c$	A1	6	or $= \frac{1}{8}\left[\frac{(2x+1)^2}{2} + \ln(2x-1)\right] + c$ CSO condone missing + c only ISW
	Total		**11**	
	TOTAL		**75**	